



Combinatorial games

Fortgeschrittenen II - Kurs „Mathematik macht Freu(n)de“

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In all the games below two players make turns one after the other. A player loses if he or she can not make a turn. Unless something different is specified, the question is to find a player who has a winning strategy (and to find a strategy if possible).

1. Two heaps with n and m stones are given. Each turn a player takes one heap and splits another one into two.[2, variation of problem 3]
2. Three tokens are placed on the three right-most cells of a $1 \times n$ -board. Each turn a player moves one token several cells to the left. A token may not jump over another token.
3. On the chessboard two rooks are given: the white one on B2 and the black one on C4. Each turn a player moves his or her rook. It is forbidden to move the rook to the same row or column with the second rook or to move it on a cell, which was occupied by a rook before.[4, 2012 Spring, Base, 10.5]¹
4. A 19×94 white board is given. Each turn a player chooses a square of any size consisting of white cells only and colors all the cells black.[3, Moscow Olympiad, 1994.8.6]
5. There are 25 tokens and a 1×25 -board. Each turn a player either puts a token on the board, or moves a previously placed token 1 cell to the right.
6. There is a 100×100 chocolate bar that has one poisoned cell with the coordinates (12, 13). Each step a player cracks the chocolate into two rectangles and eats the non-poisoned part.
7. There are 3 heaps: one with 100 stones, another with 200 stones and the third with 300 stones. Each turn a player takes one heap and splits another one into two.[1, Russian Olympiad, 1994.V.9.3]
8. A number $n > 0$ is given. There is a heap with more than n^2 stones. Each turn a player takes k stones, where k is either a prime smaller than n , or a multiple of n , or 1.[1, Russian Olympiad, 2011.IV.11.6]

¹A collection of english versions of various problems of the Tournament of Towns can be found at https://artofproblemsolving.com/community/c3239_tournament_of_towns

9. A connected bipartite graph is given. There is a red token in a vertex of one part and a blue token in a vertex of another part. Each turn a player moves his or her token by an edge to a vertex without a token. It is forbidden to make a position that already occurred in the game before (two positions are the same if the red token is in the same vertex and the blue token also). Is there a graph and a starting position of the tokens such that the first player has a winning strategy?[1, Russian Olympiad, 2018.IV.10.8]

Literatur

- [1] Archivierte Aufgaben der russischen Mathematik-Olympiade. <https://olympiads.mccme.ru/vmo/>. auf russisch (aufgerufen am 22.2.2020).
- [2] Bundeswettbewerb für Fortgeschrittene Teil 1, 2015. <https://oemo.at/0eM0/Downloads/datei/96>. (aufgerufen am 22.2.2020).
- [3] Russische Online-Aufgabensammlung. <http://www.problems.ru>. auf russisch (aufgerufen am 22.2.2020).
- [4] Turnier der Städte. <https://www.turgor.ru/en/index.php>. (aufgerufen am 22.2.2020).