2016-DSE
MATH CP
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2016

Please stick the barcode label here.

Candidate Number									

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

8.30 am – 10.45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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A030E001

SECTION A(1) (35 marks)

1. Simplify $\frac{(x^8y^7)^2}{x^5y^{-6}}$ and express your answer with positive indices. (3 marks)

$$\frac{x^{16}y^{14}}{x^5y^{-6}}$$

$$= x^{11}y^{20}$$

2. Make x the subject of the formula $Ax = (4x + B)C$. (3 marks)

$$Ax = 4Cx + BC$$

$$Ax - 4Cx = BC$$

$$x = \frac{BC}{A-4C}$$

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3. Simplify $\frac{2}{4x-5}+\frac{3}{1-6x}$. (3 marks)

$$\frac{2(1-6x)+3(4x-5)}{(4x-5)(1-6x)}$$

$$=\frac{2-12x+12x-15}{4x-24x^2-5+30x}$$

$$=\frac{-13}{-24x^2+34x-5}$$

$$=\frac{13}{24x^2-34x+5}$$ //

4. Factorize

(a) $5m-10n$,

(b) $m^2+mn-6n^2$,

(c) $m^2+mn-6n^2-5m+10n$.

(4 marks)

a) $5(m-2n)$

b) $(m-2n)(m+3n)$

c) $(m-2n)(m+3n)-5(m-2n)$

$=(m-2n)(m+3n-5)$

$m \times -2n$

$m \times +3n$

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5. In a recreation club, there are 180 members and the number of male members is 40% more than the number of female members. Find the difference of the number of male members and the number of female members. (4 marks)

let no. of male be $1.4x$ and female be x

$1.4x + x = 180$

$x = 75$.

$\therefore$ There are 75 female and 105 males.

$\therefore$ Difference $= 30$ //

6. Consider the compound inequality

$x+6<6(x+11)$ or $x \leq -5$ ..... (*).

(a) Solve (*).

(b) Write down the greatest negative integer satisfying (*).

(4 marks)

a) $x+6<6x+66$

$x>-12$

b) -1

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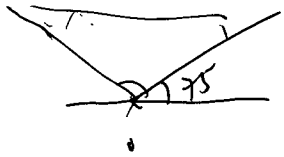
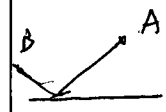


7. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(12, 75°)$ and $(12, 135°)$ respectively.



(a) Find $\angle AOB$.

(b) Find the perimeter of ΔAOB .

(c) Write down the number of folds of rotational symmetry of ΔAOB .

(4 marks)

a) $\angle AOB = 135° - 75° = 60°$

b) $AB^2 = 12^2 + 12^2 - 2(12)^2(\cos 60°)$

$AB = 12$

$\therefore$ perimeter $= 12 \times 3 = 36$ (unit)

c) 3

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8. It is given that $f(x)$ is the sum of two parts, one part varies as x and the other part varies as x^2. Suppose that $f(3) = 48$ and $f(9) = 198$.

(a) Find $f(x)$.

(b) Solve the equation $f(x) = 90$.

(5 marks)

a) $f(x) = k_1 x + k_2 x^2$, $k_1 \neq 0$, $k_2 \neq 0$

$f(3) = 3k_1 + 9k_2 = 48$ — ①

$f(9) = 9k_1 + 81k_2 = 198$ — ②

② − ① × 3

$54k_2 = 54$

$k_2 = 1$

$k_1 = 13$

$\therefore f(x) = 13x + x^2$

b) $f(x) = x^2 + 13x = 90$

$x^2 + 13x - 90 = 0$

$x = 5$ or -18

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9. The frequency distribution table and the cumulative frequency distribution table below show the distribution of the heights of the plants in a garden.

Height (m)	Frequency
0.1 – 0.3	a 2
0.4 – 0.6	4
0.7 – 0.9	b 7
1.0 – 1.2	c 9
1.3 – 1.5	15
1.6 – 1.8	3

Height less than (m)	Cumulative frequency
0.35	2
0.65	x 6
0.95	13
1.25	y 22
1.55	37
1.85	z 40

(a) Find x, y and z.

(b) If a plant is randomly selected from the garden, find the probability that the height of the selected plant is less than 1.25 m but not less than 0.65 m .

(5 marks)

a) $x = 2 + 4 = 6$ //

$z = 37 + 3 = 40$ //

$y = 37 - 15 = 22$ //

b) Req'd prop $= \dfrac{22 - 6}{40}$

$= \dfrac{2}{5}$ //

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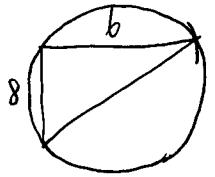

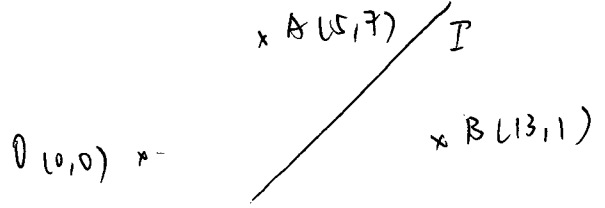

SECTION A(2) (35 marks)

10. The coordinates of the points A and B are $(5, 7)$ and $(13, 1)$ respectively. Let P be a moving point in the rectangular coordinate plane such that P is equidistant from A and B. Denote the locus of P by Γ.

(a) Find the equation of Γ. (2 marks)

(b) Γ intersects the x-axis and the y-axis at H and K respectively. Denote the origin by O. Let C be the circle which passes through O, H and K. Someone claims that the circumference of C exceeds 30. Is the claim correct? Explain your answer. (3 marks)

a) Slope of $AB = \frac{7-1}{5-13} = -\frac{3}{4}$

Slope of $\Gamma = \frac{4}{3}$

Midpt of $AB = (9, 4)$

$\therefore$ Eq^n of Γ is $y - 4 = \frac{4}{3}(x - 9)$

$y = \frac{4}{3}x - 8$

b) let $H = (h, 0)$, $K = (0, k)$

Sub H into Γ

$0 = \frac{4}{3}h - 8$

$h = 6$

$\therefore H = (6, 0)$, $K = (0, -8)$

$HK = \sqrt{6^2 + 8^2}$ (Pyth. Thm.)

$= 10$

Radius of $C = 5$ (unit)

Circumference $= 2\pi(5)$

$= 31.4$ (unit)

> 30

$\therefore$ Yes.

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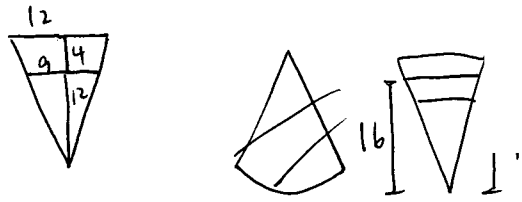

11. An inverted right circular conical vessel contains some milk. The vessel is held vertically. The depth of milk in the vessel is 12 cm . Peter then pours 444π cm^3 of milk into the vessel without overflowing. He now finds that the depth of milk in the vessel is 16 cm .

(a) Express the final volume of milk in the vessel in terms of π. (3 marks)

(b) Peter claims that the final area of the wet curved surface of the vessel is at least 800 cm^2 . Do you agree? Explain your answer. (3 marks)

a) Let the new radius of milk surface be x cm.
original radius of milk surface be y cm

$\frac{1}{3}\pi(x)^2 \times 16 - \frac{1}{3}\pi y^2 12 = 444\pi$

$16x^2 - 12y^2 = 1332$ — ①

$\frac{12}{16} = \frac{y}{x}$

$12x = 16y$ — ②

Sub ② into ① $16\left(\frac{16y}{12}\right)^2 - 12y^2 = 1332$

$y^2 = 81$

$y = 9 \quad x = 12$

∴ Final volume of milk $= \frac{1}{3}\pi 9^2 \times 12 + 444\pi$

$= 768\pi$ (cm^3)

b) Wet curved surface area

$= \pi(12)(\sqrt{12^2 + 16^2})$

$= 753.98$ (cm^2)

< 800 (cm^2)

∴ No.

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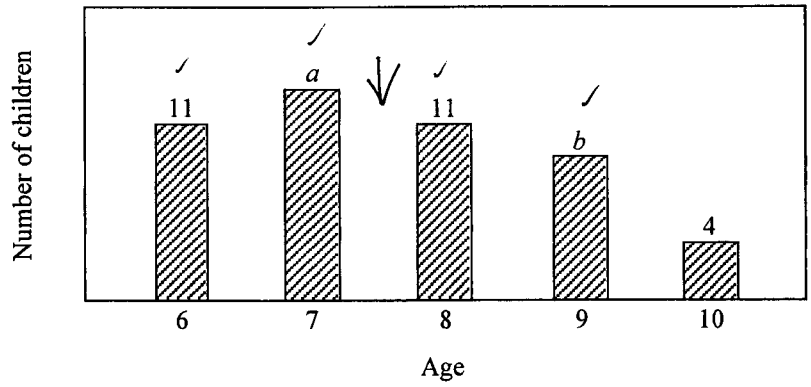
12. The bar chart below shows the distribution of the ages of the children in a group, where $a > 11$ and $4 < b < 10$. The median of the ages of the children in the group is 7.5.

Distribution of the ages of the children in the group



(a) Find a and b. (3 marks)

(b) Four more children now join the group. It is found that the ages of these four children are all different and the range of the ages of the children in the group remains unchanged. Find

(i) the greatest possible median of the ages of the children in the group,

(ii) the least possible mean of the ages of the children in the group.

(4 marks)

a) $\frac{11+a+11+b+4}{2} = 11+a$

$\therefore 26+a+b = 22+2a$

$4 = a-b$

$\because a > 11,\ 4 < b < 10$

$\begin{cases} a = 12 \\ b = 8 \end{cases}$ or $\begin{cases} a = 13 \\ b = 9 \end{cases}$

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b i) Greatest possible median
$= 8$

ii) Least possible median
$= 7$

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13. In Figure 1, ABC is a triangle. D, E and M are points lying on BC such that $BD = CE$, $\angle ADC = \angle AEB$ and $DM = EM$.

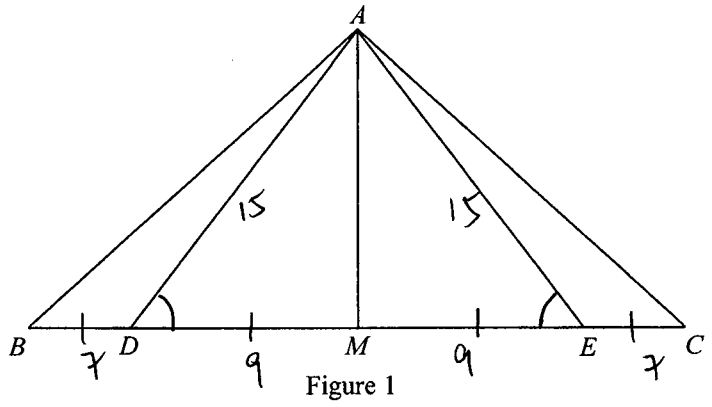

Figure 1

(a) Prove that $\Delta ACD \cong \Delta ABE$. (2 marks)

(b) Suppose that $AD = 15$ cm, $BD = 7$ cm and $DE = 18$ cm.

(i) Find AM.

(ii) Is ΔABE a right-angled triangle? Explain your answer.

(5 marks)

a) $\angle ADC = \angle AEB$ (given)

$DM = EM$ and $BD = CE$ (given)

$\therefore DB + DM = ME + EC$

$AD = AE$ (base $\angle$, isos Δ)

$\therefore \Delta ACD \cong \Delta ABE$ (SAS)

bi) $AE = AE = 15$ (corr. sides, $\cong$ s)

$\therefore \Delta ADE$ is isos Δ.

[x] $\therefore AM \perp DE$

$\therefore AM^2 + 9^2 = 15^2$ (Pyth Thm.)

$AM = 12$ (cm)//

ii) $BM = 18 + 7 = 25$ (cm)

$AB = \sqrt{12^2 + (7+9)^2}$ (Pyth. Thm.)

$= 20$ (cm).

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$AB^2 + AE^2 = 625$

$= BE^2$

$\therefore \angle BAE = 90^\circ$ (converse of Pyth. Thm.)

$\therefore$ Yes

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14. Let $p(x) = 6x^4 + 7x^3 + ax^2 + bx + c$, where a, b and c are constants. When $p(x)$ is divided by $x+2$ and when $p(x)$ is divided by $x-2$, the two remainders are equal. It is given that $p(x) \equiv (lx^2 + 5x + 8)(2x^2 + mx + n)$, where l, m and n are constants.

(a) Find l, m and n. (5 marks)

(b) How many real roots does the equation $p(x) = 0$ have? Explain your answer. (5 marks)

a) $p(-2) = p(2)$

$6(-2)^4 + 7(-2)^3 + a(-2)^2 - 2b + c = 6(2)^4 + 7(2)^3 + 4a + 2b + c$

$40 + 4a - 2b = 152 + 4a + 2b$

$b = -28$

$\therefore p(x) = 6x^4 + 7x^3 + ax^2 - 28x + c$

$(lx^2 + 5x + 8)(2x^2 + mx + n)$

$= 2lx^4 + lmx^3 + lnx^2 + 10x^3 + 5mx^2 + 5nx + 16x^2 + 8mx + 8n$

$= 2lx^4 + (lm + 10)x^3 + (ln + 5m + 16)x^2 + (5n + 8m)x + 8n$

$\therefore 2l = 6$

$l = 3$ //

$lm + 10 = 7$

$m = -1$ //

$5n + 8m = -28$

$n = -4$ //

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b) $p(x) = (3x^2+5x+8)(2x^2-x-4) = 0$

let $f(x) = 3x^2+5x+8$

$$x = \frac{-5 \pm \sqrt{5^2-4(3)(8)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{-71}}{6}.$$

$\therefore$ no real roots.

let $q(x) = 2x^2-x-4$

$$x = \frac{1 \pm \sqrt{1-4(2)(-4)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{33}}{4}$$

$\therefore$ $p(x)=0$ have 2 real roots.

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SECTION B (35 marks)

15. If 4 boys and 5 girls randomly form a queue, find the probability that no boys are next to each other in the queue. (3 marks)

Total no. of possibility $= 9!$

$= 362880$

Req'd prop. $= \dfrac{9\times8\times4\times3\times3\times2\times2\times1\times1\times2+9\times5\times3\times4\times2\times3\times2}{362880}$

$= \dfrac{3}{40}$ //

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16. In a test, the mean of the distribution of the scores of a class of students is 61 marks. The standard scores of Albert and Mary are −2.6 and 1.4 respectively. Albert gets 22 marks. A student claims that the range of the distribution is at most 59 marks. Is the claim correct? Explain your answer.

(3 marks)

let the standard deviation be σ marks.

$$-2.6 = \frac{22-61}{\sigma}$$

$$\sigma = 15.$$

let mary's mark be x

$$1.4 = \frac{x-61}{15}$$

$$x = 82$$

$\because 82 - 22 = 60$

> 59

$\therefore$ no.

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17. The 1st term and the 38th term of an arithmetic sequence are 666 and 555 respectively. Find

(a) the common difference of the sequence, (2 marks)

(b) the greatest value of n such that the sum of the first n terms of the sequence is positive. (3 marks)

a) Let common difference be d

$$666 + (38-1)d = 555$$

$$d = -3$$

$\therefore$ Common difference is -3.

b) $\frac{n}{2}[2(666) + (n-1)(-3)] > 0$

$\frac{n}{2}(1332 - 3n + 3) > 0$

$-1.5n^2 + 667.5n > 0$

$n(n-445) > 0$

$\therefore$ Greatest value of n is 445

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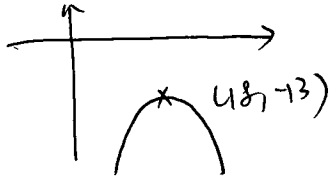

18. Let $f(x) = \frac{-1}{3}x^2 + 12x - 121$.

(a) Using the method of completing the square, find the coordinates of the vertex of the graph of $y = f(x)$. (2 marks)

(b) The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ vertically. If the graph of $y = g(x)$ touches the x-axis, find $g(x)$. (2 marks)

(c) Under a transformation, $f(x)$ is changed to $\frac{-1}{3}x^2 - 12x - 121$. Describe the geometric meaning of the transformation. (2 marks)

a) $f(x) = -\frac{1}{3}(x^2 - 36x) - 121$

$= -\frac{1}{3}(x^2 - 36x + 18^2) - 13$

$= -\frac{1}{3}(x - 18)^2 - 13$

$\therefore$ Vertex is $(18, -13)$

b) Vertex of $g(x) = (18, 0)$

$\therefore g(x) = -\frac{1}{3}(x - 18)^2$

$= -\frac{1}{3}(x^2 - 36x + 18^2)$

$= -\frac{1}{3}x^2 + 12x - 108$

c) let new $f(x)$ be $p(x)$

$p(x) = -\frac{1}{3}(x^2 + 36x) - 121$

$= -\frac{1}{3}(x + 18)^2 - 13$

$\therefore$ Vertex of $p(x)$ is $(-18, -13)$

$\therefore$ The transformation is reflect $f(x)$ along y-axis.

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19. Figure 2 shows a geometric model $ABCD$ in the form of tetrahedron. It is given that $\angle BAD = 86°$, $\angle CBD = 43°$, $AB = 10$ cm, $AC = 6$ cm, $BC = 8$ cm and $BD = 15$ cm.

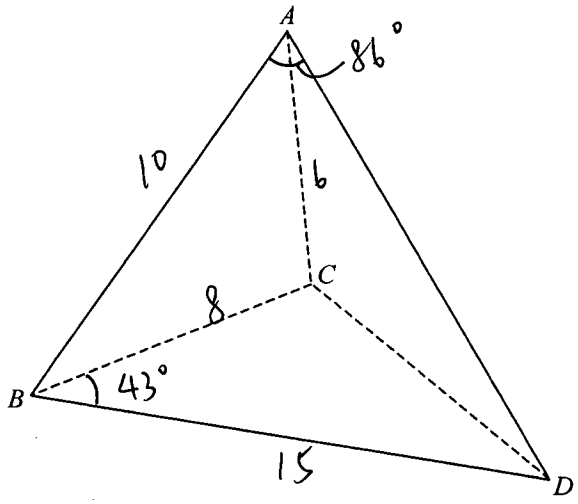

Figure 2

(a) Find $\angle ABD$ and CD. (4 marks)

(b) A craftsman claims that the angle between AB and the face BCD is $\angle ABC$. Do you agree? Explain your answer. (2 marks)

a) $\frac{15}{\sin 86°} = \frac{10}{\sin \angle ADB}$

$\angle ADB = 41.686°$

$\angle ABD = 180° - \angle ADB - 86°$ ($\angle$ sum of $\triangle$)

$= 52.3°$ //

$CD^2 = 8^2 + 15^2 - 2(8)(15)(\cos 43°)$

$CD = 10.7$ (cm) //

b) $AC^2 + BC^2 = 100$

$= AB^2$

$\therefore \angle ACB = 90°$ (Converse of Pyth. Thm.)

$\therefore$ Yes.

[X]

[Y]

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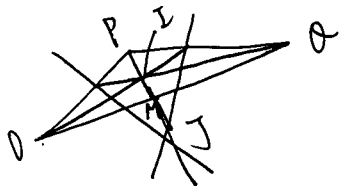

20. ΔOPQ is an obtuse-angled triangle. Denote the in-centre and the circumcentre of ΔOPQ by I and J respectively. It is given that P, I and J are collinear.

(a) Prove that $OP = PQ$. (3 marks)

(b) A rectangular coordinate system is introduced so that the coordinates of O and Q are $(0, 0)$ and $(40, 30)$ respectively while the y-coordinate of P is 19. Let C be the circle which passes through O, P and Q.

(i) Find the equation of C.

(ii) Let L_1 and L_2 be two tangents to C such that the slope of each tangent is $\frac{3}{4}$ and the y-intercept of L_1 is greater than that of L_2. L_1 cuts the x-axis and the y-axis at S and T respectively while L_2 cuts the x-axis and the y-axis at U and V respectively. Someone claims that the area of the trapezium $STUV$ exceeds $17\,000$. Is the claim correct? Explain your answer.

(9 marks)

a) $\angle OPI = \angle QPI$ (prop. of $\angle$ bisector)

let Mid-pt of OQ be M

$PM = MP$ (common side)

$\therefore$ P, I, M, J lie on same st. line.

$\therefore \angle PMO = \angle PMQ = 90^\circ$

$\therefore \triangle PMO \cong \triangle PMQ$ (ASA)

$\therefore OP = PQ$ (com. side of $\cong \triangle$)

bi) let P be $(p, 19)$

$\sqrt{(19-0)^2+(p-0)^2} = \sqrt{(30-19)^2+(40-p)^2}$

$361 + p^2 = 121 + 1600 - 80p + p^2$

$p = 17$

$\therefore P = (17, 19)$

$\therefore$ Circumcentre of $\Delta OPQ = (56, -33)$

Radius of $C = \sqrt{(-33-0)^2+(56-0)^2}$

$= 65$.

$\therefore$ Eq^n of C is $(x-56)^2+(y+33)^2 = 65^2$

$(x-56)^2+(y+33)^2 = 4225$

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bii) let eq^n of L_1 be $y = \frac{3}{4}x + s$

L_2 be $y = \frac{3}{4}x + V$

Slope of diameter of $C = -\frac{4}{3}$

let intersection point of L_1 and C be (x, y)

$$\frac{y+33}{x-56} = -\frac{4}{3}$$

$$3y + 99 = -4x + 224$$

$$3y + 4x = 125$$

$$y = \frac{125-4x}{3} \quad \text{—} (1)$$

Sub (1) into C

$$(x-56)^2 + \left(\frac{125-4x}{3} + 33\right)^2 = 4225$$

$$x^2 - 112x + 3136 + \frac{(125-4x)^2}{9} + 22(125-4x) + 1089 = 4225$$

$$x^2 - 112x + 3136 + \frac{15625 - 1000x + 16x^2}{9} + 2750 - 88x - 3136 = 0$$

$$x^2 - 200x + \frac{16x^2}{9} - \frac{1000x}{9} + \frac{15625}{9} + 2750 = 0$$

$$x = 95 \text{ or } 17$$

$$y = -85 \text{ or } 19$$

$\therefore$ intersection of L_1 and C is $(17, 19)$

L_2 and C is $(95, -85)$

$\therefore$ Eq^n of L_1 = $y - 19 = \frac{3}{4}(x - 17)$

$$y = \frac{3}{4}x + 6.25$$

Eq^n of L_2 = $y + 85 = \frac{3}{4}(x - 95)$

$$y = \frac{3}{4}x - \frac{625}{4}$$

Answers written in the margins will not be marked.

let $S=(0,s)$, $T=(t,0)$

$0=\frac{3}{4}t+6.25$

$t=-\frac{25}{3}$

$s=\frac{3}{4}(0)+6.25$

$s=6.25$

$\therefore S=(0,6.25)$ $T=(-\frac{25}{3},0)$

let $U=(u,0)$ $V=(0,v)$

$0=\frac{3}{4}u-\frac{625}{4}$

$u=\frac{625}{3}$

$v=\frac{3}{4}(0)-\frac{625}{4}$

$v=-156.25$

$\therefore U=(\frac{625}{3},0)$ $V=(0,-156.25)$

END OF PAPER

Answers written in the margins will not be marked.

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